A HIGH-FREQUENCY HIGH-PRESSURE GAS FLOW DISCHARGE AS A SLOW COMBUSTION PROCESS

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Induction heating of a gas flowing through a coil is widely used, but only the static case (induction heating of a substance at rest where all the heat is lost by conduction to the cooled walls) has been considered theoretically [1, 2]. An approximate theory is proposed here for electrodeless discharge, and the process is found to be analogous to flame propagation in a combustible mixture.

1. General picture; analogy with flame propagation. We use the parameters of the apparatus made by M. I. Yakushin at the Institute of Mechanics Problems AS USSR. Figure 1 shows the system, in which 1 is a tube, 2 is a coil, 3 is an annulus through which the gas enters, 4 is the flow of cold gas, 5 is the eddy region, 6 is the hot gas, and 7 is the emerging hot jet. The quartz tube carries air or argon at atmospheric pressure. The discharge is stabilized and displaced from the wall by tangential supply [3], so the gas flows mainly at the periphery of the tube in a spiral fashion (longitudinal velocity v about 1 m/sec). There is a reduced pressure at the axis on account of the centrifugal action of the gas, and here there are eddies together with very slow longitudinal motion. The radius of the discharge decreases as the gas flow rate increases [4] and is usually 0.4–0.8 of the tube radius.

Consider a steady-state discharge. Induction heating requires that the gas enter the discharge region adequately ionized, i.e., hot.* Early work (see [1] for survey) showed that the gas is heated by conduction from the hot layers, and the process in that respect is analogous to slow combustion, in which the combustible mixture is heated by conduction to the ignition point [5]. **

The electrical conductivity σ resembles a reaction rate in that it increases with T, especially below 8000° K, where σ is proportional to the electron concentration, while $\sigma \propto T^{3/2}$ above about 12 000° K. For a discharge we speak of a striking (ionization) temperature T_0, below which the gas behaves as almost nonconducting and screens off the magnetic field only weakly, while for $T > T_0$ the gas is heated by the induction currents and attenuates the magnetic field. We call the surface at T_0 the discharge front.

The discharge (energy release) occurs in some annular layer (effective width a) within the coil, where

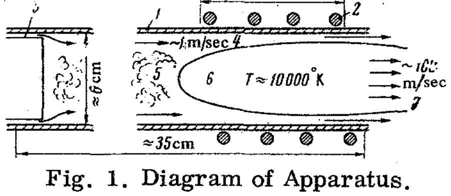

Fig. 1. Diagram of Apparatus.

a is half the thickness δ of the skin layer for penetration of the magnetic field; a is often small relative to the radius. For instance, $a = \delta/2 = c/4\pi\sqrt{\sigma\nu} = 0.15$ cm [see (3.1)] for air at atmospheric pressure with $T = 10^4$ deg K, $\sigma = 25\ \mathrm{ohm}^{-1}\mathrm{cm}^{-1} = 2.2 \cdot 10^{13}\ \mathrm{sec}^{-1}$, and $\nu = 15$ MHz, while the discharge radius R is about 2 cm.

The thickness Δ of the heated layer ahead of the discharge front is several times a. The discharge (reaction) zone forms with the heating zone the equivalent of a flame (hatched region in Fig. 2). This figure shows also the gas streamlines or rather the projection of the spirals on a diametrical plane. These lines bend at the "flame," since the heated gas expands and accelerates mainly in a direction perpendicular to the surface. The space within the discharge surface is filled with gas heated to the final temperature T_k.

This convergence on the axis raises the pressure there within the coil, and some of the hot gas may return upstream to the eddy zone, where the pressure is lower (recirculation of hot gas [3]). If the flow rate is high, a hot tongue may penetrate a long way in this fashion (all speeds are subsonic and the pressure differences are small).

The lateral discharge front is inclined to the incident gas flow in such a way that (as in a flame) the cold gas enters along the normal with a definite velocity. Analogy with the theory of combusion [5] leads us to call the normal velocity of discharge propagation u, which will be shown to be about 10 cm/sec, so the inclination of the front is small for $v \approx 1$ m/sec.

The lateral front can never be strictly parallel to the gas flow, and the gas must enter the discharge with a lateral velocity. Imagine, for example, that the front was instantaneously parallel to the flow.

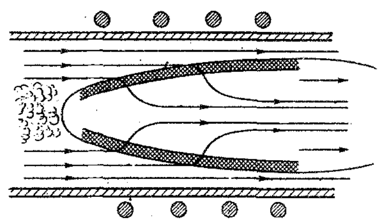

Fig. 2. Theoretical representation of the process. The "flame" is hatched.

*Relaxation processes are rapid at atmospheric pressure, and the gas can be considered as being everywhere in thermodynamic equilibrium.

**Diffusion of active reagents from the flame is a second method of preparation for ignition; the analogy here is electron diffusion. However, the diffusion is ambipolar and is slower than heat transfer because ions diffuse more slowly than do neutral particles.

The heat propagates radially, and the discharge front migrates when T_0 is reached. Heat is also carried by the flow, so it is carried to the rear part of the coil, where it later is carried off. The T_0 isotherm thus becomes inclined to the flow, and the steady state is reached when the heat input and removal come into balance, which occurs when the rising speed of gas entry into the discharge along the normal attains the velocity u.

An electrodeless discharge has some major differences from combustion. In the absence of loss, T_k is independent of the conditions for flame propagation and is determined solely by the calorific value of the mixture. For a discharge T_k is dependent on the external conditions even for a fixed rate of supply of electromagnetic energy, and the power itself is actually not known.

The main object of the theory is to calculate T_k, the flow rate for the hot gas, and the power supplied to the discharge as functions of the magnitude and frequency of the current in the coil.

2. Idealized steady-state discharge propagation. We make some simplifying assumptions about the "flame" of Figs. 2 and 3. The layer is relatively thin, so we consider it planar and one-dimensional, i.e., free from gradients along the surface. In writing the field equations we neglect the inclination of the discharge front to the coil axis and assume that the coil is an unbounded solenoid. Then vectors E and H are tangential to the surface. We also neglect radiative loss and heat loss to the layer of cold gas that keeps the discharge away from the tube wall but that does not enter the discharge. Heat transfer to the layers of cold gas that subsequently enter the discharge is not a source of loss.

We set the x-axis perpendicular to the surface of the discharge and direct it into the layer in which the energy release occurs (Fig. 3). The tangential component of the velocity in the incident oblique flow remains unchanged because there is no gradient along the surface, while the normal component v_x changes in accordance with the equation of continuity $\rho v_x = \rho_0 u$, in which ρ_0 is the density of the cold gas. The density $\rho(x)$ decreases s $T(x)$ increases roughly as $1/T$, since the pressure remains roughly constant.*

We write the energy equation bearing in mind that $dT/dt = v_x(dT/dx)$ for a gas particle in the steady state:

$$\rho_0 u c_p \frac{dT}{dx} = -\frac{dJ}{dx} + \sigma\langle E^2\rangle, \qquad J = -\varkappa\frac{dT}{dx}. \quad (2.1)$$

Here c_p is the specific heat at constant pressure, $\varkappa$ is thermal conductivity, and $\langle\rangle$ denotes averaging with respect to time over the period of oscillation of the field ($H, E \sim e^{-i\omega t}$). We direct the z-axis along the magnetic field ($H = H_z$, $E = E_y$), and then Maxwell's equations become

$$-\frac{dH}{dx} = \frac{4\pi}{c}\sigma E, \qquad \frac{dE}{dx} = \frac{i\omega}{c}H. \quad (2.2)$$

The boundary conditions for system (2.1), (2.2) follow directly from the physical formulation. The field does not penetrate deeply into the conducting plasma, while the temperature takes the steady value T_k, i.e., there is no heat flux. The gas is cold at a sufficient distance ahead of the discharge, i.e., the initial temperature $T_{-\infty} \ll T_k$, while the amplitude H_0 of the field here is real and is found as usual for a solenoid [6] as the product of the current I and turns per unit length n. There are thus five boundary conditions to be met by the solution to the four first-order equations for T, J, H, and E,**

$$J = 0,\ H = 0 \text{ for } x = +\infty,$$

$$T = T_{-\infty} \approx 0, J = 0,\ H = H_0 = \frac{4\pi}{c} In \text{ for } x = -\infty. \quad (2.3)$$

This is possible only for a particular value of u, which must be determined as a result of solving the equations (compare the situation in the theory of flame propagation).

System (2.1)–(2.3) has an integral that expresses the conservation of the total energy flux. We integrate (2.1) from $-\infty$ to x with (2.2) and (2.3) to get

$$\rho_0 u w + J + S = S_0, \quad (2.4)$$

$$w = \int_0^T c_p dT,$$

$$S = \frac{c}{4\pi}\langle EH\rangle = -\frac{c^2}{32\pi^2}\frac{1}{\sigma}\left\langle\frac{dH^2}{dx}\right\rangle. \quad (2.5)$$

Here w is the specific enthalpy and S is the flux density of the electromagnetic energy, while S_0 refers to $x = -\infty$ and is the electromagnetic power passing from the solenoid to unit discharge surface. We refer (2.4) to $x = +\infty$ to get the energy-balance equation for the layer:

$$\rho_0 u w_k = S_0, \qquad w_k = w(T_k). \quad (2.6)$$

This trivial equation relates u, w_k, and s_0, which have to be determined. We use (2.6) to put the integral of (2.4) as

$$J = \rho_0 u (w_k - w) - S \quad (2.7)$$

Then the power P supplied to the discharge and the flow rate G of heated gas are

$$P = \int S_0 dF, \qquad G = \int u dF, \quad (2.8)$$

in which the integrals are taken over the outer surface of the discharge.

3. Approximate solution. Equations (2.2) and (2.3) are closed if we know $\sigma(x)$, the distribution of the conductivity. Figure 4 shows the general form of $T(x)$ and $\sigma[T(x)]$. We replace the actual $\sigma(x)$ by a step function: $\sigma = 0$ for $x < 0$ and $\sigma = \text{const}$ for $x > 0$ (dashed line in Fig. 4). The latter must correspond to const $= \sigma(T_k) = \sigma_k$. The conductivity step is equivalent to T_0, which is essentially the simulation parameter and is naturally dependent on T_k; it is determined by substituting the

*The pressure difference across the layer is $|p_k - p_0| = |\rho_0 u^2 - \rho_k v_{xk}^2| \approx \rho_0^2 u^2/\rho_k \ll p_0$. The magnetic pressure is less than the gas pressure difference and plays no part.

**The amplitudes of E and H are complex, so the actual numbers of equations and boundary conditions are twice as great.

Table 1

amp-turns/cm	H_0, Oe	T_{ks}, 10^3 deg	T_k, 10^3 deg	T_0, 10^3 deg	w_k, kJ/g	$\sigma_k \times 10^{-13}$ sec^{-1}	$\varkappa_k \times 10^{-4}$ erg/cm sec deg	S_0, kw/cm^2	u, cm/sec	P, kW	G, cm^3/sec
						Air					
19	24	7.2	7	6.4	26	0.24	40	0.085	2.6	8.5	260
37	46	8.9	8	7.1	38	0.66	35	0.20	4.1	20	410
53	66	10.4	9	6.9	43	1.38	12	0.27	4.9	27.	490
66	76	11.0	10	7.6	48	2.18	10	0.28	4.6	28	450
72	91	11.9	11	8.8	53	2.92	13	0.36	5.2	36	520
94	119	13.0	12	10	62	3.71	18	0.54	6.9	54	680
						Argon					
4.7	5.9	7.1	7	6.4	3.65	0.41	1.9	0.0040	0.62	0.4	62
10	13	8.1	8	7.3	4.23	1.35	2.5	0.011	1.5	1.1	150
20	25	9.1	9	8.1	4.96	2.34	3.6	0.031	3.5	3.1	350
32	40	10.3	10	8.8	6.06	3.15	5.2	0.070	6.5	7.0	650
52	65	11.6	11	9.5	8.17	3.87	7.0	0.16	11	16	1100
77	97	13.0	12	10.6	11.8	4.77	12	0.31	15	31	1500

representation into the system of equations or into some equations derived from them.

Equations (2.2) and (2.3) can be solved [6] if $\sigma = 0$ or a constant. For the conductor

$$H, E \sim e^{-x/\delta}, \qquad S = S_0 e^{-x/a}, \text{ for } x > 0,$$

$$S_0 = \frac{cH_0^2}{16\pi}\left(\frac{\omega}{2\pi\sigma_k}\right)^{1/2}, \qquad a = \frac{\delta}{2} = \frac{c}{2\sqrt{2\pi\omega\sigma_k}}. \tag{3.1}$$

Ahead of the conductor

$$H = H_0, \quad S = S_0 \quad \text{for } x < 0$$

The first formula of (3.1) defines the flux of electromagnetic energy into the discharge if T_k is known; formula (2.6) then gives u.

Now we can find T (x). We substitute into (2.6) for J and S to get an equation for T (x). For $x > 0$, where $T_0 < T < T_k$, we have

$$J = -\varkappa(T)\frac{dT}{dx} = -\rho_0 u w(T) + \rho_0 u w_k (1 - e^{-x/a}). \tag{3.2}$$

It will be seen from the results of the calculations that $T_k - T_0$ is small, so (3.2) can be linearized, the more so since $\varkappa$ and c_p are not greatly dependent on T. Setting

$$\varkappa = \varkappa(T_k) = \varkappa_k, \qquad w = w_k + c_{pk}(T - T_k)$$

and integrating (3.2) with $T = T_k$ for $x = +\infty$, we get

$$T = T_k - \frac{w_k}{c_{pk}}\frac{e^{-x/a}}{1 + \Delta_k/a}, \qquad \Delta_k = \frac{\varkappa_k}{\rho_0 u c_{pk}}. \tag{3.3}$$

We apply this formula to $x = 0$, where $T = T_0$. We transform the equation via the approximation $c_{pk}(T_k - T_0) \approx w_k - w_0$ and substitute $u = S_0/\rho_0 w_k$ from (2.6), with S_0 and a replaced via (3.1), the result being

$$T_k - T_0 = \frac{c^2 H_0^2}{64\pi^2}\frac{w_0}{w_k}\frac{1}{\varkappa_k \sigma_k}. \tag{3.4}$$

This essentially expresses T_0 in terms of T_k, and its significance will become clear later on.

In the region $x < 0$, where $S = S_0$, we have from (2.4) that $J - \rho_0 uw$. To linearize $\varkappa dT/dx = \rho_0 uw$ over the wide range $0 < T < T_0$ can lead to a substantial quantitative error, but it gives at once a general indication of T (x) in this region.

Setting $\varkappa = \text{const}$, $c_p = \text{const}$, $w_p = c_p T$ and integrating subject to $T = T_0$ at $x = 0$, we get

$$T = T_0 \exp(-|x|/\Delta), \qquad \Delta = \varkappa/\rho_0 u c_p.$$

Then $\Delta \approx \Delta_k$ and is several times larger than a. The distribution T (x) in the heating zone does not affect T_k, so we will not derive it more exactly.

The equation for T_k is derived via an integral relation that is a generalization of the second integral of the system, which exists [2] in the static case. We multiply (2.7) by $\sigma(T)$ and use (2.1) and (2.5) for J and S to integrate the equation with respect to x from $-\infty$ to $+\infty$ to get

$$\int_0^{T_k} \sigma(T)\varkappa(T)\left[1 - \frac{\rho_0 u (w_k - w)}{J}\right] dT = \frac{c^2 H_0^2}{64\pi^2}. \tag{3.5}$$

In the static case ($u \equiv 0$) we get [2] instead of (3.5) that

$$\int_0^{T_{ks}} \sigma(T)\varkappa(T)\, dT = \frac{c^2 H_0^2}{64\pi^2}. \tag{3.6}$$

This relation gives* directly $T_{ks}(H_0)$, the maximum temperature far from the wall.

We substitute the J (T) implied by the approximate solution for T into the exact equation (3.5); we eliminate $\varkappa$ from (3.2) and (3.3), and bear in mind that, from (3.3) and (3.4),

$$\frac{\Delta_k}{a} = \frac{w_0}{w_k - w_0}, \tag{3.7}$$

which gives

$$J = -\rho_0 u w_0 (w_k - w)/(w_k - w_0)$$

$$\text{for } x > 0, \quad T_0 < T < T_k.$$

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*In fact, the discharge has a small but finite separation from the wall in the static case, and the wall temperature $T_1 \neq 0$, so the integral of (3.6) is taken from T_1, not from zero [2]; however, $\sigma(T)$ falls rapidly for low T, so this is of no importance in calculating the integral of (3.6)

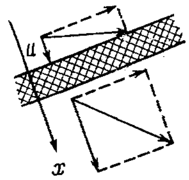

Fig. 3. Part of the "flame" layer. The velocity resolution shows the cause of streamline bending.

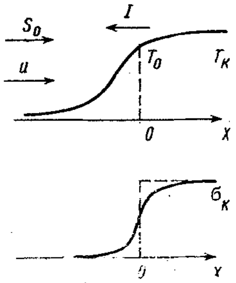

Fig. 4. Distribution of T and σ, and representation of σ.

Also, J $-\rho_0uw$ for x < 0. We substitute for J(T) into (3.5) to get

$$\int_0^{T_k} \sigma \varkappa dT + \int_0^{T_0} \sigma \varkappa \left(\frac{w_0}{w} - 1\right) dT = \frac{c^2 H_0^2}{64\pi^2} \frac{w_0}{w_k}. \tag{3.8}$$

This is combined with (3.4) to give

$$\int_0^{T_k} \sigma \varkappa dT + \int_0^{T_0} \sigma \varkappa \left(\frac{w_0}{w} - 1\right) dT = \sigma_k \varkappa_k (T_k - T_0). \tag{3.9}$$

This defines $T_0(T_k)$, which can be used with (3.8) to find $T_k(H_0)$, whereupon (2.6) and (3.1) give S_0 and u. The quantity T_k is independent of the frequency (on the basis of the assumption about the small thickness of the skin layer).

Equation (3.9) gives another view on parameter T_0. We put (3.5) in the equivalent form

$$\int_{-\infty}^{\infty} \sigma(x) S(x) dx = \frac{c^2 H_0^2}{64\pi^2}. \tag{3.10}$$

Equation (3.9) is the result of equating the integrals in (3.5) or (3.10) to the quantity obtained by approximating the integrands; in other words, T_0 (or the point at which we place the discontinuity in $\sigma(x)$) is chosen so as to keep roughly unchanged the area bounded by the weighted $\sigma(x)$ curve, the weight being $S(x)$. This choice for the step gives the correct approximation corresponding to the internal properties of the equations.

It is readily seen that this approximation corresponds to Zel'dovich's approximation in the theory of flame propagation [5]. If in (3.7) we put $\Delta_k = \varkappa_k/\rho_0 u c_{pk}$ (see (3.3)), we can represent u in a form characteristic of the thermal conduction mechanism of propagation:

$$u = \frac{\rho_k}{\rho_0} \frac{\chi_k}{a} \frac{w_k - w_0}{w_0} \quad \left(\chi_k = \frac{\varkappa_k}{\rho_k c_{pk}}\right). \tag{3.11}$$

Here χ_k is the thermal diffusivity of the gas at T_k. The power per cm^3 of gas is as follows in this approximation:

$$q = -\frac{dS}{dx} = \frac{S_0}{a} e^{-x/a} = q_{max} e^{-x/a},$$

$$q_{max} = \frac{S_0}{a} = \frac{\rho_0 u w_k}{a}.$$

We substitute the expression for a in terms of q_{max} into (3.11) and replace $w_k - w_0$ by $c_{pk}(T_k - T_0)$ to get that

$$u = \frac{1}{\rho_0 w_k}\left[2\varkappa_k q_{max}(T_k - T_0)\left(\frac{1}{2}\frac{w_k}{w_0}\right)\right]^{1/2}.$$

This formula agrees with Zel'dovich's for the flame speed apart from a factor $(w_k/2w_0) \sim 1$ (the calorific value of the mixture equals the final enthalpy).

4. Results. Table 1 gives results for air and argon at atmospheric pressure and ν = 15 MHz; the electrical conductivity for air was calculated from [1], while $\sigma(T)$ for argon was taken from [7]. The thermal conductivities were taken from [8] (air was equated to nitrogen).

The value of $T_k - T_0$ is found to be fairly small, which gives another major analogy with combustion. Reaction rates vary rapidly with T, so the ignition temperature is only a little below the final temperature of the products [5]. The heat produced in the reaction zone is transferred almost entirely by conduction to the initial mixture in order to heat it to the ignition point.

The analogous situation here means that the balance equation (2.7) differs little from J + S = 0 for the release zone, and the latter corresponds to the static condition, with w_0 not very different from w_k, and the second integral in (3.8) is substantially less than the first, so (3.8) does not differ very greatly from (3.6) and T_k is not far below the maximum temperature T_{ks} corresponding to the static case with the same In (Table 1).

The last two columns of the table give P and G, which have been calculated without allowance for the difference in the conditions at the various parts of the discharge surface (see (2.8)), namely as P = $= S_0F$, G = uF, and F = $2\pi RL$, where R is the mean radius of the discharge ring and L is its length, which is obviously close to the length of the coil. We assumed R = 2 cm (neglecting the power dependence of R [4]), L = 8 cm, and F = 100 cm^2. The results are in general agreement with experiment. The P corresponding to the highest T (10 000 to 12 000°) may have been underestimated on account of radiative loss.

5. Lines of development. The above solution may be used as a first approximation, e.g., by extending a procedure [2] for the static case; numerical calculations are here essential, of course. The problem is easily formulated and readily solved approximately for cylindrical geometry (the gas enters radially into an infinite discharge cylinder). This takes some account of the effects of the axial symmetry, which are important at low frequencies, where the thickness of the skin layer is comparable with the radius. We can refine P and G by applying (2.8) with allowance for the variation in H_0 over the discharge surface because the field of a short coil differs from that of an infinite solenoid. Here one should use experimental evidence on the shape of the surface.

A detailed theory requires a knowledge of the flow distribution at the entry to the hot zone, which, in particular, would enable one to determine R as a function of G [4]. A general picture of the spiral flow can be obtained from the approximate theory of centrifugal pumping [9], which for our conditions gives that all the gas flows with a constant axial velocity in a peripheral layer whose thickness is 0.08 of the tube radius, the rest of the tube being filled by an eddy. However, this theory makes no allowance for viscosity and the reduced speed at the wall, which appear to be very important for the tube lengths usually employed. The viscosity makes the treatment much more complicated. Here experimental evidence should be used in conjuction with the calculations.

Also, one should consider the gas dynamics of the flow in the hot zone, since without this one cannot determine the recirculation of the hot gas. The radiation from the hot gas should also be considered, as well as the inward heat flux due to radiation from the energy-release zone.

The above model forms the basis of a program of experimental studies.

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